Semidefinite Programming for Stochastic Wireless OFDMA Networks

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1 Introduction

Resource allocation such as maximizing link capacity or minimizing power consumption in a wireless OFDMA network are commonly formulated as mathematical programs [1]. These programs usually involve random variables in the input data. In this paper, we propose a (0-1) stochastic quadratic formulation. The study is made on the basis of an OFDMA quadratic model [4] in which a probabilistic constraint based approach is considered [2]. Then, a semidefinite programming (SDP) relaxation is derived to solve the stochastic quadratic model. The paper is organized as follows: Section 2 presents the stochastic quadratic formulation. Section 3 presents the SDP relaxation. Finally, section 4 concludes the paper.

2 Probabilistic formulation

We consider an OFDMA network composed by a base station (BS) and several mobile users. The BS has to assign a set of N sub-carriers to a set of K users using a modulation size of $c \in \{1, \ldots, M\}$ bits in each sub-carrier. The goal is to minimize the total power consumption in the network. We consider the following probabilistic constrained model [4]:

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$$
\begin{aligned}
\text{SQIP0: } \min_{\{x_{k,n}, y_{n,c}\}} \quad & \textstyle\sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} P_{k,n}^{c} x_{k,n} y_{n,c} && (1)\\
\text{st:} \quad & \mathbb{P}\{\textstyle\sum_{n=1}^{N} x_{k,n} \left[\sum_{c=1}^{M} c \cdot y_{n,c}\right] \geq R_k\} \geq (1-\alpha_k), \quad \forall k && (2)\\
& \textstyle\sum_{k=1}^{K} x_{k,n} \leq 1 \quad \forall n && (3)\\
& \textstyle\sum_{c=1}^{M} y_{n,c} \leq 1 \quad \forall n && (4)\\
& x_{k,n}, y_{n,c} \in \{0,1\} && (5)
\end{aligned}
$$

Here, the objective function represents the total power consumption. The first constraint corresponds to a chance constraint where α_k is the risk to be taken for each user k. In this model, we consider separated chance constraints. The second constraint imposes that each sub-carrier should be assigned to only one user at a time while the third one imposes that each sub-carrier must use one integer modulation size. The decision variables are given by $x_{k,n}$ and $y_{n,c}$, respectively. We assume that R_k are random variables with joint probability distribution H. Let us consider the case where H is concentrated in the finite number of points also called scenarios $R_k = (r_{k,1}, ..., r_{k,l}, ..., r_{k,L_k})$ with probabilities $p_{k,l}$ such that $\sum_{l=1}^{L_k} p_{k,l} = 1, p_{k,l} \geq 0, \forall k$.

Then, the problem (1)-(5) can be reformulated as follows [2]:

$$
\begin{aligned}
\text{SQIP1: } \min_{\{x_{k,n}, y_{n,c}\}} \quad & \textstyle\sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} P_{k,n}^{c} x_{k,n} y_{n,c} && (6)\\
\text{st:} \quad & \textstyle\sum_{n=1}^{N} x_{k,n} \left[\sum_{c=1}^{M} c \cdot y_{n,c}\right] \geq r_{k,l} \quad \forall l \in \Gamma_k, \forall k && (7)\\
& \textstyle\sum_{l \in \Gamma_k} p_{k,l} \geq 1-\alpha_k \quad \forall k && (8)\\
& \textstyle\sum_{k=1}^{K} x_{k,n} \leq 1 \quad \forall n && (9)\\
& \textstyle\sum_{c=1}^{M} y_{n,c} \leq 1 \quad \forall n && (10)\\
& x_{k,n}, y_{n,c} \in \{0,1\} && (11)
\end{aligned}
$$

Constraints (8) mean that we have to choose a subset Γ_k of scenarios such that the sum of the probabilities of this subset is greater than $(1-\alpha_k)$. For this subset, the bit rate constraints will be active and valid, whereas for the scenarios not in this subset, the constraints are not activated.

This problem can be reformulated by introducing the auxiliary binary variable $\varphi_{k,l}$ for each observation $l = 1 : L_k, \forall k$ as follows:

$$
\varphi_{k,l} = \begin{cases} 0 \text{ if } \ l \in \Gamma_k \\ 1 \text{ otherwise} \end{cases} \tag{12}
$$

This yields the following problem:

$$
\text{SQIP2: } \min_{\{x_{k,n}, y_{n,c}, \varphi_{k,l}\}} \quad \textstyle\sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} P_{k,n}^{c} x_{k,n} y_{n,c} \tag{13}
$$

$$
\begin{aligned}
\text{st:} \quad & \textstyle\sum_{n=1}^{N} x_{k,n} \left[\sum_{c=1}^{M} c \cdot y_{n,c}\right] \geq r_{k,l} - \mathcal{M}\varphi_{k,l} \quad \forall k, l = 1 : L_k & (14)\\
& \textstyle\sum_{l=1}^{L_k} p_{k,l}\varphi_{k,l} \leq \alpha_k \quad \forall k & (15)\\
& \textstyle\sum_{k=1}^{K} x_{k,n} \leq 1 \quad \forall n & (16)\\
& \textstyle\sum_{c=1}^{M} y_{n,c} \leq 1 \quad \forall n & (17)\\
& x_{k,n}, y_{n,c}, \varphi_{k,l} \in \{0, 1\} & (18)
\end{aligned}
$$

where $\mathcal{M}$ is an arbitrary number such that

$$
\mathcal{M} \geq \max_{k,l}\{r_{k,l}\} + 1 \tag{19}
$$

This problem is a quadratic optimization problem with binary variables. This quadratic problem is NP-hard, and so is its stochastic formulation. In this case, we seek lower bounds using strong relaxations, namely SDP relaxations.

3 Semidefinite relaxation

In order to write a SDP relaxation for SQIP2, we define the (0-1) vector $z^T = (x_{1,1}, \cdots, x_{1,N}, \cdots, x_{K,1}, \cdots, x_{K,N}, y_{1,1}, \cdots, y_{1,M}, \cdots, y_{N,1}, \cdots, y_{N,M}, \varphi_{1,1}, \cdots, \varphi_{1,L_1}, \cdots, \varphi_{K,1}, \cdots, \varphi_{K,L_k})$. Then, let Z be a symmetric positive semidefinite matrix defined as:

$$
Z = \begin{pmatrix} zz^T & z \\ z^T & 1 \end{pmatrix} \succeq 0 \tag{20}
$$

We can construct symmetric matrices $\mathcal{P}$ for the objective function in (13), $U_{k,l}$ for constraints in (14) and V_k for constraints in (15). We propose the following SDP relaxation for SQIP2:

$$
\begin{aligned}
\text{SSDP2} : \min_{Z} \quad & Trace(\mathcal{P}Z) & (21)\\
\text{st: } & Trace(U_{k,l}Z) \geq r_{k,l} \quad \forall k, l = 1 : L_k & (22)\\
& Trace(V_k Z) \leq \alpha_k \quad \forall k & (23)\\
& Trace([ex_n][ex_n]^T Z) \leq 1 \quad \forall n & (24)\\
& Trace([ey_n][ey_n]^T Z) \leq 1 \quad \forall n & (25)\\
& Trace(\zeta_{k,n}^{c} Z) \geq 0 \quad \forall k, n, c & (26)\\
& diag(zz^T) = z & (27)\\
& Z \succeq 0 & (28)
\end{aligned}
$$

In this model, $[ex_n]$ and $[ey_n]$ are coefficient vectors for constraints in (16) and (17) according to vector z. Thus, the rank-1 matrices we construct with these vectors are used to strength our SDP relaxation [3]. The symmetric matrices $\zeta_{k,n}^c$ for all $\{k, n, c\}$ are used to have positive values in matrix Z only in the positions where $\{\mathcal{P}_{i,j}, i < j\}$ is positive, this is, in the entries of $\mathcal{P}$ where we put the elements $\{P_{k,n}^c, k, n, c\}$ from (13). Finally, constraint (27) together with constraint (28) form a relaxation constraint for the condition of $z_i \in \{0, 1\}$ for all i. The last constraint also imposes the condition on matrix Z to be positive semidefinite. Our SDP relaxation is tighter than the linear program (LP) we obtain by applying Fortet linearization method [5] to SQIP2 as shown by our preliminary results.

4 Conclusions

In this paper, we proposed a stochastic quadratic formulation for wireless OFDMA networks. To this purpose, we considered an OFDMA quadratic model [4] in which probabilistic constraints are added by using the approach of [2]. Finally, a SDP relaxation is derived. Numerical results are given.

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