Wide - sense nonblocking $\log_d(N, 0, p)$ networks

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1 Introduction

One of the common architectures that are used in high-speed photonic and electronic switching networks is the $\log_d(N, m, p)$ switching network [1,2]. In this paper we consider a special case of such network, $\log_d(N, 0, p)$ network. Usually, the architecture is accompanied with the control algorithm that works online, i.e. after arrival of a connection request, a path through the network is assigned that connects the input and output of the new connection [3,4]. A switching network is wide sense nonblocking (WSNB) if a new call is always routable as long as all previous requests were routed according to a given routing algorithm [5]. WSNB switching networks were first introduced by Beneš [6] for symmetric 3−stage Clos network. He proved that $C(n, m, 2)$ is WSNB if and only if $m \geq \left\lfloor \frac{3n}{2} \right\rfloor$.

Here we introduce an auxiliary *path graph* of switching network such that the number of required planes is the chromatic number of the path graph. From the structure of the path graphs it follows that the minimal p is $d^{\left\lfloor \frac{n}{2} \right\rfloor} - 1$ for every n. Furthermore, our analysis implies that at least p planes are needed regardless of the control algorithm used.

A special case of this result was proved for a particular algorithm and even n (for $d = 2$ in [7] and for general d in [8]). For other cases only bounds in terms of so-called non-blocking conditions were known [5,9,10].

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2 Basic definitions and notations

The basic conponents of switching network are crossbar switches and links which connect switches. The $\log_d(N, 0, p)$ switching network consist of p copies of $\log_d(N, 0, 1)$, called the planes. Each plane contains $n \cdot d^{n-1}$ switches divided into n stages. In each stage there are d^{n-1} switches and each switch has d inputs and d outputs. Inputs and outputs are numbered $0, 1, ..., N-1$, $N = d^n$, from top to bottom, and stages are numbered $0, 1, ..., n$ from left to right. Examples of $\log_2(8, 0, 1)$, $\log_2(16, 0, 1)$, and $\log_3(27, 0, 1)$ switching networks are given on figures 1, 2, and 3 respectively.

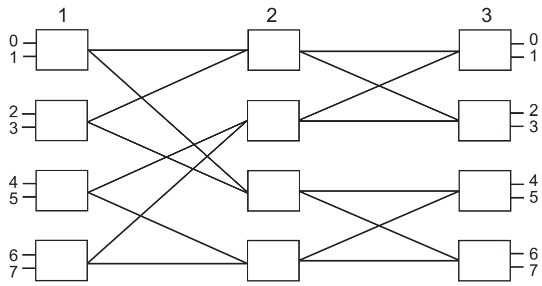

Fig. 1. Example of switching network for $n = 3$ and $d = 2$.

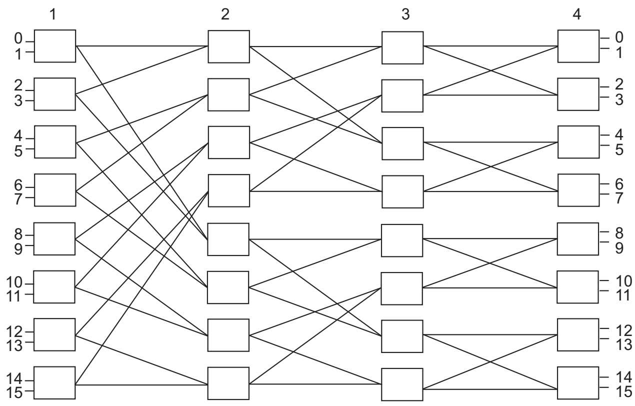

Fig. 2. Example of switching network for $n = 4$ and $d = 2$.

Definition: The **path graph** $G_P(n, d)$ of a switching network $\log_d(N = d^n, 0, 1)$ is the graph with a vertex (i, j) for every connection $\langle i, j \rangle$ in the switching network. Two vertices (i_1, j_1) and (i_2, j_2) are adjacent in G_P if connections $\langle i_1, j_1 \rangle$ and $\langle i_2, j_2 \rangle$ share an interstage link in switching network.

For example in figure 2 the connections $< 0, 0 >, < 1, 5 >$ share the link between stages 1 and 2, so they are adjacent. The connections $< 0, 0 >, < 2, 9 >$ are independent because they do not share a link.

Proposition. The **path graph** $G_P(n, d)$ is in the worst case isomorphic to

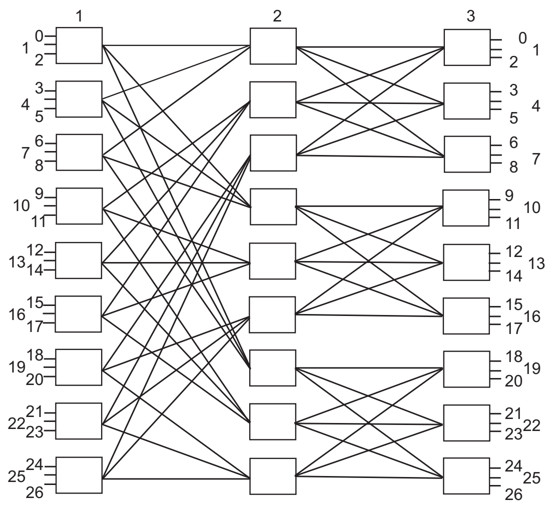

Fig. 3. Example of switching network for $n = 3$ and $d = 3$.

the strong product $(K_d \square K_d) \boxtimes K_{d^{k-2}}$ when $n = 2k - 1$, $k \in \mathbb{N}$ and isomorphic to the d^k copies of K_{d^k}, when $n = 2k$, $k \in \mathbb{N}$.

Proof ommited in the abstract.

Proposition. The number of planes p needed to make $\log_d(N, 0, p)$ switching network a WSBN is equal to the chromatic number of the path graph.

$$p = \chi\left(G_P(n, d)\right)$$

(Proof ommited in the abstract.) Idea of proof: by construction of the path graph, the vertices corresponding to independent connections are independent and two vertices are connected when the connections share a link. Hence any subset of connections can be can be divided into $p = \chi\left(G_P(n, d)\right)$ independent sets and each of them can be realized in a different plane. On the othere hand, it is easy find a set of connections that forms a clique, so it can not be realized in less than p planes without blocking.

Remark. From details of our construction one can find a formula which assigns the color (the plane) on which it should be realized, so there is no complicated algorithm for assigning the planes needed. Compare to [7].

Recall that the chromatic number of the strong product of graphs is less than or equal to product of chromatic numbers of factors. Equality holds when chromatic number of both factors is equal to their clique number [11].

Hence we have

Theorem: The $\log_d(N, 0, p)$ switching network, $n = \log_d N$, is WSNB if and only if

$$p \geq \begin{cases} d^k, & if\ n = 2k,\ k \in \mathbb{N} \\ d^{k-1}, & if\ n = 2k-1,\ k \in \mathbb{N}. \end{cases}$$

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